

# **How to Make Your Kids Stop Saying “6-7”**

*Or: How to Transcend Brain Rot*

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*A general-audience exploration of what happens when a transcendental constant is observed through integer arithmetic, and why the result speaks in sixes and sevens.*

Formal addendum included for mathematical readers.

## Part I — The Nuclear Option

There is, as it turns out, a nearly foolproof way to get your kids to stop saying “6-7” and it is not what you think it is, it is not banning it or scolding them or pretending you are somehow too sophisticated to have noticed, which never works anyway because children can smell pretence the way certain breeds of dog can smell seizures, with an accuracy that is frankly unsettling and a response time that makes you wonder who is really in charge of the household.

No, the actual move is much worse than any of those things.

You take an interest in it.

A real interest. The kind of interest only a parent can take, slow and patient and just a little too sincere, the kind that wraps itself around whatever was once light and fun and socially alive and slowly, gently, without malice but with tremendous gravitational force, crushes it into something educational. You do not argue with it. You do not dismiss it. You say something like “oh that is actually really interesting, did you know there is a reason sixes and sevens keep showing up in mathematics” and then you just keep talking.

And if you really want to make absolutely sure the thing dies you do not stop at the surface, you do not do the reasonable thing of offering one or two fun facts and then moving on with your life, no, you take it all the way down to a kind of number so strange and so fundamental that mathematicians had to invent a special word for it, and that word is *transcendental*, and we will get to what that means but first I want to be honest with you about something.

You do not need to remember any math to read this.

I am not going to assume you remember algebra. I am not going to assume you know what a function is or what “mod” means or what any of the symbols do. If you are a person who last thought about mathematics sometime around eighth grade and have since gotten on with the business of living a life and raising children and wondering why those children keep saying “6-7” at the dinner table, then you are exactly the person this is for. I will explain everything. Step by step. In order. Using words.

And by the end you will either have killed the meme forever or accidentally discovered something genuinely beautiful about numbers, and honestly either outcome is acceptable.

At that point one of two things happens. Either the child’s eyes glaze over and the phrase never returns, buried under the sheer weight of parental enthusiasm, or the child starts asking questions. And if they start asking questions you have a completely different and arguably much better problem on your hands because now you are no longer killing a trend, you are doing mathematics together, which is a thing that almost never happens at a dinner table and is frankly more nutritious than anything you were serving.

This is, to be clear, a joke.

But only partly.

Because sixes and sevens really do show up. Not as a coincidence, not as a vibe, not as the kind of pattern you see when you stare at ceiling tiles for too long. As an actual fact about what happens when you do a very specific thing to numbers. The sixes and sevens are built into the structure of a particular number, a number you may have heard of called pi, or rather its less famous but arguably more natural sibling called tau, and the reason they show up has to do with the fact that tau belongs to a very exclusive and very strange club of numbers called transcendental numbers.

That is the real subject of this paper. Transcendence. Everything else, the tricks with digits, the loops, the patterns, all of it, is scaffolding to help you see what transcendence does when it collides with ordinary counting.

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## **Part II — A Parent’s Guide (No Math Required)**

This is the part you can actually use. At a table, in a car, in the middle of some repetitive chant that is slowly hollowing out your soul. The explanation has to work out loud. It has to make sense to a child. And it has to make sense to you first, which means I need to start from the very beginning, and I mean the very beginning, because in my experience the problem with most math explanations is not that they are wrong but that they start about four steps further along than they should.

### **Numbers that never end**

Start here.

“You know how some numbers are nice and tidy? Like 3, or half, or 7.5? You can write them down and they stop.”

Your child knows this even if they would not say it this way. Some numbers fit neatly on paper. Three is three. Half is half. Seven and a half is seven and a half. Done. Finished. The pencil can stop moving.

“But some numbers are not like that. Some numbers go on forever. You start writing them out and they never stop and they never repeat. You could write for the rest of your life and you would never finish.”

This is genuinely strange when you think about it. We are so used to the idea that a number “is” something definite that the notion of a number whose digits just keep going with no pattern and no end is actually kind of wild. But such numbers exist. They are everywhere. And they are not broken or unfinished, they are just a fundamentally different kind of thing.

The most famous one is pi. You have heard of pi. It is about 3.14159 and it goes on forever and it has to do with circles and you may dimly remember something about it from school and that is more than enough.

## **Pi and tau: the half trip and the whole trip**

Here is the single most important thing to understand about pi and you can say this to your child directly:

“Pi tells you how far it is halfway around a circle. If you draw a circle where the distance from the centre to the edge is 1, the distance halfway around that circle is pi, about 3.14. The distance all the way around is about 6.28.”

That is it. That is what pi is. The halfway-around-a-circle number.

Now here is the thing. If you are talking about things that go all the way around, things that loop, things that come back to where they started, then halfway around is a slightly odd unit of measurement. It would be like measuring laps around a track but only counting half-laps and then having to double everything at the end. It works, it is technically fine, but it is one extra step that does not need to be there.

So some people, and I am one of them, prefer to use a number called tau. Tau is just two times pi. It is about 6.28318. And what tau measures is the distance all the way around that same circle, the one with radius 1. The full trip. One complete loop. Back where you started.

“Tau is just the full-circle version of pi. Pi gets you halfway. Tau gets you home.”

For the rest of this paper I will use tau instead of pi because everything we are about to look at is about full cycles and returns and things coming back, and it is just cleaner to talk about the whole trip than to keep saying “well take the half trip and double it.”

## **Degrees, radians, and a surprisingly cool fact about shapes**

Now you might be thinking “hang on, I already have a perfectly good way of measuring angles and circles, it is called degrees, and it works fine.” And you are right. You probably remember that a right angle is 90 degrees, a straight line is 180 degrees, and a full circle is 360 degrees. You might even remember that the angles inside a triangle always add up to 180 degrees, which is one of those facts that sounds like it could maybe not be true until you tear the corners off a paper triangle and line them up and they really do make a straight line every single time.

Degrees work. Nobody is taking degrees away from you.

But there is another way to measure the same thing, and it is called radians, and the reason mathematicians prefer it is that instead of using the somewhat arbitrary number 360 (which was chosen by the ancient Babylonians because they liked multiples of 60, which is a fine reason to organise a calendar but a slightly odd reason to build a measurement system around), radians measure angles in terms of how far you have actually travelled around the circle itself.

In radians, a full circle is tau. That is it. One full trip around, tau. Half a trip, pi. A quarter turn, a right angle, is tau divided by 4, which is pi divided by 2. The angle

system and the circle number are the same thing. There is no extra 360 sitting around for historical reasons. The number just is the angle.

And once you see it this way, some things that were already cool in degrees become even cooler.

You may remember that the inside angles of a triangle always add up to 180 degrees. In radians, 180 degrees is pi. So the inside angles of every triangle in the universe always add up to exactly pi. That is already a nice sentence.

But here is the one that might genuinely surprise you. Take any closed shape, any polygon, it does not matter how many sides it has. A triangle, a square, a pentagon, a hexagon, anything, as long as it does not cross over itself. Now instead of looking at the inside angles, look at the outside angles, the ones you would turn through if you were walking along the edge and turning at each corner.

Those exterior angles always add up to 360 degrees. Always. No matter what the shape is. No matter how many sides. A triangle, a thousand-sided polygon, it does not matter. (If the shape has dents in it, some of those turns go the other way, like turning left instead of right, but the total still comes out to one full rotation.)

And in radians, 360 degrees is tau. One full turn.

“The inside angles of a triangle add up to pi. The outside angles of any shape add up to tau.”

That is not a coincidence. When you walk all the way around any closed shape, turning at every corner, you end up facing the same direction you started in. You have made one full rotation. One full turn. Tau. It does not matter whether the shape has three sides or three hundred, because the total turning is always one complete revolution.

This is why mathematicians like radians, and why they especially like pi and tau. These numbers are not just measuring circles in the abstract. They are baked into the geometry of every shape, every angle, every rotation. Degrees give you 180 and 360, which are fine but do not tell you anything deep. Radians give you pi and tau, which are the actual structural constants that geometry runs on.

And these numbers, these constants that underpin all of this, turn out to be transcendental. Which brings us to the really strange part.

## **What makes these numbers special**

Here is where it gets genuinely interesting and you can tell this to your child too.

Some forever-numbers, even though they go on without end, are still the answer to a simple question. The square root of 2, for example, goes on forever, but it is still the answer to “what number times itself gives you 2?” It never ends but at least you can describe what it is in a short sentence.

Pi is not like that. Tau is not like that.

They are not the answer to any simple equation. Not “what times itself gives you something” or “what cubed gives you something” or any question of that form, no matter how complicated you make the question, no matter how many “times itself”

or “cubed” or “to the fourth power” operations you stack up. They are permanently outside the reach of that kind of arithmetic.

Numbers like this are called **transcendental**. The word literally means “climbing beyond.” They transcend, they climb past, the entire system of equations that ordinary algebra can produce.

“There are some numbers that are so wild, so untameable, that you cannot pin them down with any equation. No matter how complicated an equation you write, these numbers will never be the answer. Mathematicians call them transcendental.”

That is not a metaphor. That is a theorem. It was proved in the 1800s and it is one of the most remarkable facts in all of mathematics.

And pi, and therefore tau, are the most famous members of that club.

### **Euler and the most beautiful equation**

If the child is still listening, and this is the point where you find out if you have a runner on your hands, you can introduce the person who connected all of this together.

“There was a mathematician called Leonhard Euler who was so wildly productive that they actually had to stop naming things after him because otherwise half of mathematics would just be called Euler’s Something.”

This is barely an exaggeration. The man published more mathematics than almost any human being who has ever lived, he went blind in his later years and it barely slowed him down, and if anything the output seemed to increase, which is the kind of biographical detail that makes you suspect the universe is sometimes just showing off.

Euler discovered that four of the most important numbers in all of mathematics, numbers that seem to have nothing to do with each other, are secretly connected by one equation. And if you write it using tau it looks like this:

$$“e^{i\tau} = 1”$$

I know that looks like someone dropped a bowl of symbols on the floor. So let me translate.

There is a number called  $e$ , it is about 2.71828, and it is another transcendental number that shows up whenever things grow or shrink or compound or decay. There is a number called  $i$ , which is an imaginary number, and I realise that sounds fake, like something a math teacher made up to annoy you, but it is real in the sense that it is genuinely useful and shows up everywhere in physics and engineering and you do not need to understand it fully right now. And there is tau, our full-circle number.

Euler’s discovery is that if you combine these three things in exactly the right way, you get 1. Exactly 1. The most important transcendental numbers in existence, combined with the strangest number in all of arithmetic, somehow conspire to produce the simplest number there is.

“Three of the weirdest numbers in mathematics, combined in one equation, give you 1. That’s it. Just 1.”

You do not need to understand why this works. You need to understand that it is real, that it is considered by many mathematicians to be the most beautiful equation ever written, and that it means these transcendental numbers are not isolated weirdnesses floating around in the number line. They are connected to each other and to the geometry of circles in a way that is deep and fundamental and, once you see it, very hard to unsee.

### **What any of this has to do with 6-7**

Right. So. You may be wondering what any of this has to do with your child’s annoying phrase. Fair question.

Here is the connection and I promise it is a real one.

When you take ordinary counting numbers, 1, 2, 3, 4 and so on, and you divide them by tau, and then you do two very simple things to the result, something extraordinary happens. Those two simple things are:

**First**, you throw away the decimal part. This is called taking the “floor” and it is exactly as unsophisticated as it sounds. 3.7 becomes 3. 12.99 becomes 12. 100.001 becomes 100. You just keep the whole number and dump the rest.

**Second**, you count in a loop of 9. Imagine a clock that only goes up to 9 instead of 12. Once you pass 9, you start over at 0. So 10 becomes 1, 11 becomes 2, 18 becomes 0, 19 becomes 1, and so on.

That is it. Those are the operations. Divide by the circle number. Throw away the decimal. Count in a loop of 9.

And when you do this, starting from 0 and going up, the results break into blocks. And the blocks are always length six or length seven. Always. Not sometimes, not mostly, not “about six or seven give or take.” Always exactly six or exactly seven. No fives, no eights, no anything else. Just sixes and sevens, forever, in a sequence that never repeats but never stops being structured.

The reason this happens is precisely because tau is transcendental. If tau were a nice tidy fraction, the blocks would eventually settle into a simple repeating pattern and that would be the end of it. But because tau transcends ordinary arithmetic, because it cannot be captured by any finite equation, the underlying division never lands cleanly on an integer and the raw sequence drifts forever. And yet, and this is the key thing, when you observe that drift through the floor function and the loop of 9, those operations throw away just enough information that the pattern you can actually see does come back in bounded windows. The full infinite sequence is not periodic. But finite stretches of it return with extraordinary precision. That tension between the raw source never repeating and the observed pattern coming back anyway is where all the interesting structure lives.

“So when you say ‘6-7,’ there is actually a real pattern in mathematics where sixes and sevens show up, and it is because of one of the most untameable

numbers in existence.”

That is the connection. That is the punchline. A transcendental number, observed through the simplest possible integer operations, speaks in sixes and sevens. It does not know about the meme. The meme does not know about it. But the rhyme is real.

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## **Part III — Going Deeper (Still No Math Degree Required)**

If Part II was enough to kill the meme, you can stop here and go back to your life with a clean conscience and a quiet household. But if you or your child want to know what the pattern actually looks like, this is where we go deeper.

Everything in this section is real. It has been computed, verified, and checked. I am going to describe it in plain language because the formal version is at the end for people who want it, but the structure itself is genuinely remarkable and you do not need equations to appreciate why.

### **The first phrase**

When you run the divide-by-tau, take-the-floor, count-in-loops-of-9 machine starting from zero, the first seven blocks come out with lengths:

7, 6, 6, 7, 6, 6, 6.

That is not random. It has a shape. A pair of long blocks at the start, then shorter ones, then a long one again, then a tail. It is rhythmic. It sounds like something. If you were clapping it out it would have a groove to it, the way a drum pattern does, and that is not an accident, the mathematical structure of this thing has genuine similarities to the way rhythm works in music.

This first group of seven blocks covers the numbers from 0 to 43. That is 44 numbers total, which already introduces a small but important quirk: the last number is 43 but the total count is 44 because you started at zero. That off-by-one issue between “where does it end” and “how many are there” turns out to be a recurring theme throughout the entire pattern, like a running joke that keeps showing up in new contexts.

### **The 710 scaffold**

If you zoom out and look at the numbers from 0 to 709, those 710 numbers contain exactly 113 blocks. Eighty-one of them are length 6 and thirty-two are length 7. They are not scattered randomly. They are arranged in a specific architecture: fifteen repetitions of that first seven-block phrase, interrupted by two shorter break phrases, like a song with verses and a bridge. The last block ends at position 709, and position 710 is where the next cycle begins.

And 710 and 113 are not random numbers. They are the best way to approximate tau using a fraction. 710/113 is extremely close to tau, just like the famous approximation 355/113 is extremely close to pi. (And notice: 710 is just 2 times 355, exactly as tau is 2 times pi. The scaffold knows what it is made of.)

## **The full cycle at 6,390**

709 is where the block pattern first wraps after 710 positions cleanly, and 710 is where it starts again. But it is not the full story. After 113 blocks, something has shifted by 5 in the loop-of-9 system, and since 5 and 9 do not share any common factors, you need to go through the whole process nine times before everything resets completely. Nine times 710 is 6,390.

Over those 6,390 positions (0 through 6,389), every digit from 0 to 8 appears as a block value exactly 113 times and occupies exactly 710 positions. Everything is perfectly balanced. Every piece of the scaffold shows up in every other piece of the scaffold. It is like a building where every beam is load-bearing and every room connects to every other room.

And at this level, something else shows up that does not exist at the 710 level. If you add up all the block values, you get 4,068. If you add up all the starting positions in their loop-of-9 form, you also get 4,068. If you add up all the ending positions, you also get 4,068. Three completely different sums landing on the same number. And 4,068 is not just any number. It is what you get when you take the total block-sum, which is 25,560 (that is 6,390 times 4), and divide it by tau and take the floor. The same tau that generated the whole pattern in the first place shows up governing the relationship between the sums. And 4,068 factors as  $2^2 \times 3^2 \times 113$ , the same 113 that has been running the block counts from the very beginning. This is not a coincidence. It is the system's own arithmetic echoing back at a higher level.

## **The pattern comes back (and then it doesn't)**

Here is my favourite part.

Remember that tau is transcendental, so if you just look at the raw division  $n/\tau$ , it never lands back on the same values. The digits keep drifting forever. But here is the trick: we are not looking at the raw division. We are looking at what happens after the floor function and the mod-9 loop have had their way with it. Those two operations throw away the fine decimal detail and keep only the coarse staircase shape. And at the level of that coarse shape, the pattern can and does come back.

At one very specific large number, 1,308,519 copies of the 6,390-block later, the observed pattern returns to itself point by point. Exactly. Not approximately. Every single position in the 6,390-window matches. The underlying division is still drifting, it always is, but the floor and mod operations cannot see the difference because the drift has not yet accumulated enough to push anything over the next threshold.

And it keeps doing that for 25 repetitions. The pattern comes back 25 times, perfectly, in a row.

Then, on the 26th time, it finally breaks. And where does it break? At exactly three positions, spaced 710 apart. The same 710 that built the scaffold in the first place.

Even the failure is disciplined. Even when the pattern finally stops working, it stops working on the same grid that made the pattern exist. It is like a machine that, when it eventually wears out, breaks along the seams that held it together.

## **The big number**

The next important scale is 73,996,200. I want to be careful here because big numbers have a way of making people sound either mystical or insane and I would like to avoid both. This number is not the secret of the universe. It is the next level of the hierarchy, the span at which the block pattern, the value cycle, and the column sums all agree on a common return point. The last position in that span is 73,996,199, and the next cycle begins at 73,996,200, following the same terminal-versus-span pattern that has been running through the whole structure.

When you divide 73,996,200 by tau, you get a number extremely close to an integer, 11,776,861. And that integer is a prime number, a number that cannot be broken into smaller factors. The pattern at this scale has a prime sitting exactly where the hierarchy says a boundary marker should be.

## **Primes at the boundaries**

Speaking of primes. The first block phrase ends at position 43. Prime. The 710-block ends at position 709. Prime. The 6,390-block ends at position 6,389. Prime. The big return at 73,996,200 ends at position 73,996,199. Prime.

Not every important number in the hierarchy has a prime at its boundary. Some do not, and that selectivity is actually part of what makes it interesting. If every number had a prime at the door it would feel like a trick. Instead, the most structurally important levels tend to have primes and the others do not, and nobody yet knows the exact rule that separates them.

## **Why this matters**

It would be entirely reasonable at this point to ask “so what.” A number that goes on forever produces a pattern that goes on forever. Is that really so surprising?

And the answer is yes, actually, it is, because the pattern is vastly more structured than it has any right to be. A transcendental number is, in some deep sense, the most unruly kind of number that exists. It cannot be captured by any equation. Its digits never repeat. It lives outside the reach of algebra itself. And yet, when you observe it through the simplest possible integer operations, the thing you see through that window is an architecture of blocks and phrases and cycles and returns and even graceful failures that is more organised than most things humans build on purpose. The raw number is untameable. The view through the floor and the loop is shockingly disciplined.

That is the thing about transcendence. It does not mean chaos. It means a kind of order that is too complex for the algebraic system to describe but not too complex for the arithmetic system to see. You just have to look at it the right way.

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## **Part IV — The Punchline**

So here is what happened.

You started out trying to kill a stupid trend. That was the whole mission. Not to discover something about the nature of numbers, not to learn what “transcendental” means, not to spend an amount of time that you will never publicly admit thinking about why a circle number that goes on forever produces blocks of six and seven when you divide counting numbers by it and throw away the decimals. You were trying to make the house quieter.

And then, because the world is occasionally very rude, you pulled on one dumb thread long enough that it turned out to be attached to something real. Not because “6-7” was secretly a sacred utterance. Not because children on the internet accidentally discovered number theory. And certainly not because every stupid trend contains hidden wisdom, most of them do not, most of them are just cheap loops with good transmission properties and the best thing you can say about them is that they are efficiently compressed.

But some things survive as repetition and repetition is one of the oldest structural facts there is and the moment you stop treating the thing purely as annoying background noise and start asking “wait, why does that keep coming back” you are in a different game entirely, a game that begins with digital roots and loops and ends, if you are not careful, somewhere in the deep architecture of transcendental constants.

You can say it simply.

Brain rot is repetition severed from understanding. Mathematics is repetition re-joined to understanding.

That is the entire arc.

Children do not like baggage. Memes do not like context. Trends do not like parents with notebooks.

That is why this works.

So yes, explain it to them. Explain that there is a kind of number so wild it escapes every equation. Explain that tau is one such number. Explain that when you force it through the simplest possible counting operations the whole thing shatters into sixes and sevens. Explain that those sixes and sevens have a rhythm and the rhythm has a structure and the structure has levels and the levels have returns and even the returns eventually fail and the failures land on the same grid that built the returns

in the first place. Explain all of it calmly and precisely and with the sort of parental seriousness that makes a room go spiritually cold.

The trend will stop.

Or one of you will accidentally become the sort of person who can no longer see a stupid repeated thing without wondering what kind of number is hiding behind it.

That is not exactly a cure.

But it is, in my view, an improvement.

## Appendix — For the Mathematicians

Everything above was written for a general reader. What follows is the formal statement. The jokes stop here.

### A.1. Definition

Let  $\tau = 2\pi$ . Define the sequence

$$f(n) = \left\lfloor \frac{n}{\tau} \right\rfloor \bmod 9, \quad n \in \mathbb{N}, n \geq 0.$$

Equivalently,  $f(n) = \lfloor n\tau^{-1} \rfloor \bmod 9$ . The sequence lives in  $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ . The phase origin  $n = 0$  is structural: it aligns indexing with the interpretation of  $n/\tau$  as completed full-turn units.

### A.2. Generating mechanism

The transcendence of  $\tau$  (Lindemann-Weierstrass) implies no nonzero integer period  $P$  satisfies  $P/\tau \in \mathbb{Z}$ . The staircase  $a(n) = \lfloor n/\tau \rfloor$  has first difference  $\Delta a(n) \in \{0, 1\}$ . Reducing modulo 9 gives  $f(n) = a(n) \bmod 9$ . The sequence stages three operations: continuous scaling by  $\tau^{-1}$ , truncation via floor, and residue collapse mod 9. Crucially, while the raw scaled sequence  $n/\tau$  never exhibits finite periodic closure, the floored and reduced sequence  $f(n)$  can and does produce exact pointwise returns over bounded windows, because the floor and mod operations discard sufficient fine-grained information for the observed pattern to reset before the accumulated irrational drift crosses the next carry threshold.

### A.3. Proved results

**Block decomposition.**  $f(n)$  breaks into maximal constant runs of length 6 and 7 only. Since  $1/7 < 1/\tau < 1/6$ , the inter-jump gaps of  $a(n)$  are confined to  $\{6, 7\}$ .

**Base phrase.**  $\{7, 6, 6, 7, 6, 6, 6\}$ , spanning  $n = 0$  to  $n = 43$  (terminal index 43, span 44).

**710-cycle architecture.** Over  $0 \leq n \leq 709$ : 113 blocks (81 of length 6, 32 of length 7), arranged as  $8 + \text{break} + 7 + \text{break}$  where each break phrase is  $\{7, 6, 6, 6\}$ . The continued-fraction convergent  $710/113 \approx \tau$ .

**Sturmian spacing.** Within one 710-cycle, the 32 seven-blocks follow spacing  $(3, 4)^8 \cdot 4 \cdot (3, 4)^7$ , a Sturmian word on  $\{3, 4\}$  with slope  $32/113$ .

**6390-cycle value closure.**  $113 \bmod 9 = 5$ ,  $\gcd(5, 9) = 1$ , so nine 710-shifts required:  $6390 = 9 \times 710$ . Over  $0 \leq n \leq 6389$ : 1017 blocks (729 of length 6, 288 of length 7). Each residue  $0, \dots, 8$  appears 113 times, occupying 710 positions.

**Column-sum identities at 6390.** Block-value sum = start-index-mod-9 sum = stop-index-mod-9 sum = 4068. Block-sum (value  $\times$  length) =  $25,560 = 6390 \times 4$ .  $\lfloor 25,560/\tau \rfloor = 4068$ . These do not hold at 710.

## Prime factor skeleton.

$$\begin{aligned} 710 &= 2 \times 5 \times 71, & 113 &\text{ prime,} \\ 6390 &= 2 \times 3^2 \times 5 \times 71, & 1017 &= 3^2 \times 113, \\ 25,560 &= 2^3 \times 3^2 \times 5 \times 71, & 4068 &= 2^2 \times 3^2 \times 113, \\ 102,240 &= 2^5 \times 3^2 \times 5 \times 71, & 16,272 &= 2^4 \times 3^2 \times 113. \end{aligned}$$

Irreducible cores  $\{2, 5, 71\}$  (cycle-length side) and  $\{113\}$  (block-count side) persist at every level, with powers of 2 accumulating as the hierarchy climbs. The entire scaffold rests on four primes: 2, 3, 5, 71, and 113.

**Exact finite-window return.** Define

$$y_q(n) = \left\lfloor \frac{n + 6390q}{\tau} \right\rfloor - 113 \left\lfloor \frac{n + 6390q}{710} \right\rfloor, \quad 0 \leq n < 6390.$$

At  $q_2 = 1,308,519$ :  $y_{q_2}(n) - 113 = y_0(n)$  for all  $n$ . Persists for 25 multiples. First failures at multiple 26:  $\{4593, 5303, 6013\}$ , spaced by 710.

**102,240-cycle drift.**  $f(n)$  vs  $f(n + 102,240)$  over  $[0, 6389]$ : six mismatches at  $\{2463, 3173, 3883, 4593, 5303, 6013\}$ , all spaced by 710. Same carry-threshold mechanism as the exact-return failures.

## A.4. Observed (computationally verified, not proved from first principles)

**Meta-return scale.**  $73,996,200 = 710 \times 104,220 = 6390 \times 11,580 = 25,560 \times 2895$ . Division:  $73,996,200/\tau \approx 11,776,861.0000165$ ; 11,776,861 is prime.

**Wave confirmation.** Detuned waves with  $r_a = 113\tau/710$ ,  $r_b = \tau^{-1}$ : interference returns at  $6390 \times h \approx 73,996,200$ ,  $h \approx 11,580$ .

**Prime chain.**  $710/\tau = 113.00000959524569\dots$ ; reading left to right: 11 prime, 113 prime,  $\lfloor 710 \times 10^6/\tau \rfloor = 113,000,009$  prime.  $\lfloor 71 \times 10^n/\tau \rfloor$  prime at  $n = 7, 262, 3933$ .  $\lfloor 73,996,200 \times 10^n/\tau \rfloor$  prime at  $n = 0, 5, 74, 193, 282, 775$ .

**Decimal-tail ratios.** For  $710 \times 10^y/\tau$  at  $y = 6, 261, 3932$ : tail ratios  $4/11, 33/14, 7/6$  with product 1. Exponent gaps 255 and 3671 match prime-hit exponents.

**Terminal primes.** 43, 709, 6389, 73,996,199 all prime. Non-primes:  $25,559 = 61 \cdot 419$ ,  $102,239 = 19 \cdot 5381$ .

## A.5. Open problems

Whether the prime-adjacent terminal pattern admits a causal explanation. Whether the hierarchy persists for other transcendental constants. Whether phrase architecture derives directly from continued-fraction partial quotients. Whether the arithmetic and wave pictures are functorially related. Whether the generalised family  $f_k(n) = \lfloor n\tau^{-k} \rfloor \bmod m$  exhibits analogous structure for  $k > 1$  or prime moduli  $m$ .

## A.6. Formal core

$$f(n) = \lfloor n\tau^{-1} \rfloor \bmod 9, \quad n \geq 0.$$

A transcendental source, observed through a bounded coding, produces an exact discrete architecture: 6/7 block decomposition, Sturmian spacing, hierarchical closures at 710 and 6390, exact finite-window return, structured drift, and a larger meta-return scale that reappears across arithmetic, wave, and decimal representations.