Domain Tau

Complete Analysis of the Tau Block Pattern

Timothy Solomon
Independent Researcher, London
contact@timothysolomon.com

March 26, 2026 *(Preprint)*

Abstract

We study the discrete structures that emerge when integer arithmetic—specifically the floor function and modular reduction—acts on the transcendental constant $\tau = 2\pi$. The function $\mathrm{mod}(\lfloor n/\tau \rfloor, 9)$ generates a hierarchy of nested cycles: a 710-number cycle containing exactly 113 blocks with lengths drawn from $\{6, 7\}$ in a self-similar pattern $\{7, 6, 6, 7, 6, 6, 6\}$ interrupted by two symmetric phase breaks; a 6390-number complete value cycle; and super-cycles at 25560 and 102240 numbers preserving this architecture at scale.

We identify a family of prime-generating functions of the form $\lfloor c \cdot 10^n/\tau \rfloor$ for $c \in \{1/\tau,\ \tau,\ 71,\ 73{,}996{,}200\}$ that produce verified primes at specific exponents. The repeating decimal tails of $\lfloor 710 \cdot 10^y/\tau \rfloor$ encode the period of 1/71 and stand in clean rational ratios (4/11, 33/14, 7/6) across scales $y = 6, 261, 3932$.

In the complex-exponential extension $\sin(n^{i\tau^k})$, all orbits converge to $\sin(1)$, and the critical crossing points are identified as $e^{\pi/2}$, e^{π} (Gelfond's constant), and $e^{3\pi/2}$—integer multiples of $e^{\tau/4}$—revealing that the dynamics are governed by quarter-turn exponentials of τ in the complex plane.

The average block length of the 710-cycle is $710/113 \approx \tau$, a self-referential signature: the transcendental constant reappears as the mean of the discrete structure it generates. We frame these phenomena not as consequences of Diophantine approximation but as properties of the interaction between discrete operators and transcendental constants—what is here termed *Symbolic Necessity*: the structure that arises when a formal system acts on a quantity it operationally requires but can never internally resolve.

Keywords: tau, transcendental constants, floor function, modular arithmetic, prime generation, Sturmian sequences, symbolic necessity, discrete–transcendental interaction
MSC 2020: 11A63, 11B85, 11J81, 11Y11, 37E10

Contents

1 Preamble: Discreteness Over Transcendence

This document records a body of research into what happens when discrete arithmetic—the floor function, modular reduction, integer projection—acts directly on a transcendental constant it can never capture. The central object of study is the function

$$f(n) = \mathrm{mod}\big(\lfloor n/\tau \rfloor,\ 9\big),$$

and the structures that fall out of it: a 710-number cycle containing exactly 113 blocks, a self-similar phase pattern $\{7, 6, 6, 7, 6, 6, 6\}$ with two symmetric breaks, nested super-cycles at 6390, 25560, and 102240 numbers, and a family of prime-generating functions anchored to these same scales.

The thesis here is *not* Diophantine approximation. It would be easy to observe that $710/\tau \approx 113.00000096$ and attribute the near-integer to the classical convergent $355/113 \approx \pi$, which is famously accurate for its denominator size. But that framing treats the discrete structure as a side effect of approximation quality—it says the rationals chase the irrationals well, and the rest is noise. This research asks a different question. It starts with $\lfloor\cdot\rfloor$ and mod as *operators*—hard quantisation and finite-group projection applied to a transcendental—and asks what structure the interaction itself produces. The near-integer is not the explanation; it is the phenomenon to be explained.

By the Lindemann–Weierstrass theorem, τ is transcendental: no algebraic equation over $\mathbb{Q}$ has τ as a root. Integer arithmetic cannot construct it. But integer arithmetic can *act on* it, and when it does, the result is not noise. It is 113 blocks. It is a base pattern with phase breaks at positions 56 and 109. It is prime numbers at scales 710, 6390, 73,996,200. The coherence is in the dance between the discrete and the transcendental—not in either partner alone.

This is one expression of what the author terms **Symbolic Necessity** ($\odot$): τ as something the discrete system operationally *requires* but can never fully *resolve* from within. The block pattern, the cycle hierarchy, and the prime-generating behaviour are what that irresolution looks like from the integer side.

2 Structural Insights and Identifications

Several constants and relationships embedded in the numerical data deserve explicit identification, connecting the system to known mathematical structures.

2.1 The Settle Point Is $\sin(1)$

Throughout the n-dependent sequence algebra (Section 13), all points in the system $\sin(n^{i\tau^k})$ converge to the value 0.84147098. This is

$$\sin(1) = 0.84147098480789\ldots$$

the image of the multiplicative identity under the most fundamental periodic function. The system's attractor is not an arbitrary constant; it is the sine of unity.

2.2 The Crossing Points Are Powers of $e^{\tau/4}$

The three critical crossing points identified in the convergence analysis are not arbitrary:

Crossing	Value	Identity
First imaginary crossing	$N_{\text{firstCross}} = 4.81047738\ldots$	$e^{\pi/2} = e^{\tau/4}$
First real crossing	$N_{\text{SecondCross}} = 23.14069263\ldots$	$e^{\pi} = e^{\tau/2}$ (Gelfond's constant)
Period completion ratio	$Q \approx 111.3178$	$e^{3\pi/2} = e^{3\tau/4}$

The system's imaginary-line crossing, real-line crossing, and period completion are spaced at integer multiples of $e^{\tau/4}$. The dynamics of $\sin(n^{i\tau^k})$ are governed by the exponential of quarter-turns of τ in the complex plane. The period completion occurs at

$$N_{\text{SecondCross}} \times Q = e^{\tau/2} \cdot e^{3\tau/4} = e^{5\tau/4},$$

itself a clean multiple of the base unit $e^{\tau/4}$.

2.3 The Repeating Tail at $y = 6$ Contains $1/71$

The 35-digit repeating tail at $y = 6$,

$$16901408450704225352112676056338028,$$

encodes the decimal period of $1/71$, which is exactly 35 digits: $01408450704225352112676056338028169\ldots$ This is structurally inevitable: since $710 = 10 \times 71$, the function $\lfloor 710 \cdot 10^y/\tau \rfloor$ probes the decimal structure of $71/\tau$, and the remainder cycles with the period of the reciprocal of 71. The prime 71 is not incidental to the pattern; it is woven into the repeating decimal fabric of the output.

2.4 The Average Block Length Self-Refers to τ

The average block length of the 710-cycle is

$$\frac{710}{113} \approx 6.28318\ldots \approx \tau.$$

The mod-9 structure of $\lfloor n/\tau \rfloor$ produces blocks whose average length converges back to τ itself. This self-reference—the transcendental constant reappearing as the mean of the discrete structure it generates—is a signature of the discrete–transcendental interaction at the heart of this work.

2.5 $355/113$ as Phenomenon, Not Explanation

The rational approximation $355/113 = 3.14159292\ldots \approx \pi$ is accurate to seven significant figures and is present throughout: $710 = 2 \times 355$, and 113 is the block count. But in this framework, $355/113$ is not the *cause* of the pattern's coherence—it is one of its most visible *symptoms*. The question is not "why does $355/113$ approximate π so well?" (a question about continued fractions). The question is: "what does the integer lattice *see* when it looks at τ through the floor function?" The answer is 113 blocks, phase-locked cycles, and primes—and $355/113$ is the rational shadow that the lattice casts.

3 Introduction to Tau

3.1 Definition of Tau

Tau (τ) is defined as the ratio of a circle's circumference to its radius:

$$\tau = \frac{C}{r} = 2\pi \approx 6.28318.$$

3.2 Mathematical Foundations of Tau vs Pi

1. Primitive Measurement. The defining property of a circle is the set of points equidistant from a centre: $C = \{(x, y) \mid \sqrt{(x-h)^2 + (y-k)^2} = r\}$. The constant relating this defining measure to circumference is $\tau = C/r$; the diameter-based constant $\pi = C/2r$ introduces an additional factor. Thus τ is the primitive constant.

2. Number Theory. The field extensions $\mathbb{Q}(\tau)/\mathbb{Q}$ and $\mathbb{Q}(\pi)/\mathbb{Q}$ are identical, but τ requires fewer additional factors to express fundamental circle relationships. Its minimal polynomial over $\mathbb{Q}$ is the same as that of 2π.

3. Group Theory. The circle group S^1 has period τ: the fundamental homomorphism $\theta \mapsto e^{i\theta}$ from $\mathbb{R}$ to S^1 has kernel $\tau\mathbb{Z}$, making τ the primitive generator.

4. Information Theory. Using τ eliminates the factor of 2 in numerous formulas. By Kolmogorov complexity arguments, τ yields the shorter description.

5. Subgroup Containment. $\langle\tau\rangle$ properly contains $\langle\pi\rangle$ in the additive group of $\mathbb{R}$, since $\pi = \tau/2 \in \langle\tau\rangle$ but $\tau \notin \langle\pi\rangle$.

Aspect	**Argument**
Circle Definition	$\tau = C/r$ directly connects circumference to the circle's defining property.
Group Theory	Kernel of $\mathbb{R} \to S^1$ is $\tau\mathbb{Z}$; τ is the primitive generator.
Multiplicative Relations	$\tau^{-1} < \pi^{-1}$, giving τ precedence in natural ordering.
Natural Parametrisation	$\gamma(t) = (r\cos t, r\sin t)$ has period τ.
Fourier Analysis	Base period τ eliminates factor 2 in Fourier series.
Differential Equations	Harmonic motion: $T = \tau/\omega$ is more direct than $T = 2\pi/\omega$.
Radian Measure	A complete circle spans exactly τ radians: 1-to-1 mapping.
Complex Analysis	$e^{i\tau} = 1$: one complete rotation directly.

4 Base Function and Block Formation

4.1 Core Function

For any natural number n:

$$f(n) = \operatorname{mod}\big(\lfloor n \cdot \tau^{-1} \rfloor,\ 9\big)$$

This returns an integer 0–8, forming blocks of consecutive numbers sharing the same value.

4.2 Block Properties

Each block has: (1) Block ID, (2) Block Integer (value 0–8), (3) Block Length ($\in \{6, 7\}$), (4) Block Start n, (5) Block Stop n, (6) Block Sum (Length × Integer), (7) Start n mod 9, (8) Stop n mod 9.

4.3 Block Formation Example

ID	Integer	Length	Start n	Stop n	Sum	Start mod 9	Stop mod 9
0	0	7	0	6	0	0	6
1	1	6	7	12	6	7	3
2	2	6	13	18	12	4	0
3	3	7	19	25	21	1	7
4	4	6	26	31	24	8	4
5	5	6	32	37	30	5	1
6	6	6	38	43	36	2	7

5 The 710-Cycle Complete Analysis (113 Blocks)

5.1 Length Pattern Structure

Phase Pattern (Blocks 0–112, $n = 0$–709): the base pattern $\{7, 6, 6, 7, 6, 6, 6\}$ repeats with two interruptions across 113 blocks spanning 710 numbers.

First Phase (Blocks 0–59, $n = 0$–376): 60 blocks, 377 numbers. The core pattern (Blocks 0–55) contains 8 complete sequences of $\{7, 6, 6, 7, 6, 6, 6\}$. The *First Break* (Blocks 56–59, $n = 352$–376) inserts an extra $\{7, 6, 6, 6\}$ group (4 blocks, 25 numbers), then returns to the base pattern.

Second Phase (Blocks 60–112, $n = 377$–709): 53 blocks, 333 numbers. The core pattern (Blocks 60–108) contains 7 complete sequences. The *Second Break* (Blocks 109–112, $n = 685$–709) mirrors the first: an extra $\{7, 6, 6, 6\}$ group, then return.

5.2 Integer Value Progression

- Complete sequence progresses through all values 0–8
- Integer values cycle independently of the length pattern
- Total sum of Block Integers = 4068

5.3 Length Distribution

For the complete 710-cycle: 81 blocks of length 6, 32 blocks of length 7, totalling 113 blocks. Sum of lengths = 710; average block length = $710/113 \approx 6.2832 \approx \tau$.

6 Important Sums and Their Relationships

6.1 Column Sums

1. **Block Integer Column**: Sum = $4068 = \lfloor 25560/\tau \rfloor$
2. **Block Length Column**: Total = 6390
3. **Block Sum Column**: Sum = $25560 = 4 \times 6390$

4. **Mod 9 Columns**: Both Start and Stop n mod 9 sum to 4068

6.2 Tau Ratio Connections

$$\begin{aligned} 710/\tau &\approx 113.00000959524569 && \text{(block count)} \\ 6390/\tau &\approx 1017.0000863572112 && \text{(complete value cycle)} \\ 25560/\tau &\approx 4068.000345428845 && \text{(appears in multiple sums)} \\ 102{,}240/\tau &\approx 16{,}272.0013817 \end{aligned}$$

7 Complete Value Cycle (6390 Numbers)

Total 1017 blocks: 729 of length 6 ($= 81 \times 9$) and 288 of length 7 ($= 32 \times 9$). Each value 0–8 appears exactly 710 times; contains 9 complete 710-cycles; Block Integer sum = 4068.

8 Prime Factor Relationships

8.1 Critical Numbers

Pattern numbers (built from 71):

$$\begin{aligned} 710 &= 2 \times 5 \times 71 \\ 6390 &= 2 \times 3^2 \times 5 \times 71 \\ 25560 &= 2^3 \times 3^2 \times 5 \times 71 \\ 102240 &= 2^5 \times 3^2 \times 5 \times 71 \end{aligned}$$

Tau-ratio numbers (built from 113):

$$\begin{aligned} 113 &= 113 \quad \text{(prime)} \\ 1017 &= 3^2 \times 113 \\ 4068 &= 2^2 \times 3^2 \times 113 \\ 16272 &= 2^4 \times 3^2 \times 113 \end{aligned}$$

8.2 Key Relationships

$$\begin{aligned} 6390 &= 9 \times 710 \\ 25560 &= 4 \times 6390 = 36 \times 710 \\ 102240 &= 4 \times 25560 = 16 \times 6390 = 144 \times 710 \\ 73{,}996{,}200 &= 2895 \times 25560 = 104{,}220 \times 710 \end{aligned}$$

9 Pattern Preservation at Scale

9.1 Block Pattern Scaling

- Every 710 numbers: length pattern repeats (709 is prime)
- Every 6390 numbers: complete value cycle (6389 is prime)
- Every 25560 numbers: super-cycle (25559 is not prime)
- Every 102240 numbers: full cycle (102239 is not prime)
- After 102240: CODA pattern cycle of 6390 numbers

9.2 Shift Points in the 102240-Cycle

Shifts occur at regular 710-intervals starting from index 2463:

n	**Index**	**ΔIndex**
104703	2463	—
105413	3173	710
106123	3883	710
106833	4593	710
107543	5303	710
108253	6013	710
108963	6723	710
109673	7433	710
110383	8143	710
111093	8853	710
111803	9563	710
112513	10273	710
113223	10983	710
113933	11693	710
114643	12403	710

9.3 Large Scale Behaviour

$$73{,}996{,}200/\tau = 11{,}776{,}861.000016505\ldots$$

and $\lfloor 11{,}776{,}861.000016505\ldots \rfloor = 11{,}776{,}861$ is **prime**.

Massive-scale cycles begin at $n = 665{,}953{,}020$; $n = 1{,}331{,}906{,}040$; $n = 1{,}997{,}859{,}060$; etc.

10 Prime at Scale

10.1 Core Variables

$$a = 710, \quad y = 6, \quad b = 10^y, \quad k = ab, \quad n = k/\tau, \quad x = \lfloor n \rfloor.$$

10.2 Summary of All Prime Candidates

Found in $\lfloor \tau^{-1} \cdot 10^n \rfloor$, $\lfloor \tau \cdot 10^n \rfloor$, and $\lfloor 71 \cdot 10^n/\tau \rfloor$ for $n = 1$ to 9999. Note: the exponents n in the 71-family correspond to $y + 1$ in the repeating-tail analysis because $710 \times 10^y = 71 \times 10^{y+1}$.

Family	**Exponents yielding primes**
$\lfloor \tau^{-1} \cdot 10^n \rfloor$	$n = 71,\ 90,\ 155$
$\lfloor \tau \cdot 10^n \rfloor$	$n = 344,\ 382,\ 521,\ 3779,\ 5754$
$\lfloor 71 \cdot 10^n/\tau \rfloor$	$n = 7,\ 262,\ 3933$
$\lfloor 73{,}996{,}200 \cdot 10^n/\tau \rfloor$	$n = 0,\ 5,\ 74,\ 193,\ 282,\ 775$

10.3 Repeating Tail Patterns in $\lfloor 710 \cdot 10^y/\tau \rfloor$

When $y = 6$. After the initial digits `159154942253521126760563380028`, the repeating tail emerges:

`16901408450704225352112676056338028` (repeating)

When $y = 261$. After the initial digits:
159154943091895335768883763372514362034459645740456448747667344058896797634226535090113802766253085956072842726757958036892911846114578652877967410731699839229239966

The repeating tail emerges:

46478873239436619718309859154929577 (repeating)

When $y = 3932$. The full initial string (3932 digits) is preserved in the appendix. The repeating tail:

19718309859154929577464788732394366 (repeating)

This last repeating tail persists up to $y = 9999$.

10.4 Ratios Between Repeating Tails

Labelling the three tails as a ($y = 6$), b ($y = 261$), c ($y = 3932$):

$$a/b \approx 0.\overline{36} = 4/11$$
$$b/c \approx 2.\overline{357142857142} = 33/14$$
$$c/a \approx 1.1\overline{6} = 7/6$$

Clean ratios of small integers—suggesting fundamental structure, not coincidence.

10.5 Primes from the 710/τ Chain

$$710/\tau \approx 113.0000095952\ldots$$

From this chain: 1 (primish), 11 (prime), 113 (prime), 113,000,009 (prime), and 1,177,686,100,001 (prime).

11 Comprehensive Overview of the Tau-Wave System

11.1 Core Wave Functions

$$W_a = r_a \sin(x \cdot r_a^{-1}), \qquad r_a = \frac{113}{710\,\tau^{-1}} \approx 1.00018\pi$$
$$W_b = r_b \sin(x \cdot r_b^{-1}), \qquad r_b = \tau^{-1}$$
$$W_c = r_a \sin(x \cdot r_a^{-1} + 6390h)$$
$$W_d = r_b \sin(x \cdot r_b^{-1} + 6390h)$$

The parameter h controls phase shifting; $h = 0$ creates perfect alignment.

11.2 Wave Synchronisation Properties

- Perfect synchronisation at intervals of 710 units
- Waves return to synchronisation after $h \approx 11{,}580$ cycles
- $6390 \times h \approx 73{,}996{,}200$: a complete phase-return cycle

11.3 Prime-Generating Properties

$\lfloor 71 \cdot 10^n/\tau \rfloor$ yields primes at $n = 7$, $n = 262$, and $n = 3933$. Additionally, $\lfloor 73{,}996{,}200/\tau \rfloor = 11{,}776{,}861$ is prime, with $73{,}996{,}200 = 104{,}220 \times 710$.

11.4 Connection to Complex Analysis

The insight $\sin(x) = \sin(e^{i\tau})$ provides geometric interpretation: wave functions as projections of complex rotations; synchronisation points as rotational alignments; a geometric realisation of prime number sieves.

11.5 Proposed Connection to the Riemann Hypothesis

The tau-wave system may provide a geometric realisation of what RH describes analytically:

- Synchronisation points may correspond to resonances in spectral decomposition
- The interference pattern $(W_a - W_b) - (W_c - W_d)$ may act as a continuous Sieve of Eratosthenes
- The prime-generating function could reveal structure related to zeta zeros

11.6 Parameterised Transformation

$$F(s,t) = \frac{(1-t) \cdot W_b(n) + t \cdot n^{-s}}{n^{t \cdot \mathrm{Re}(s)}}$$

where s is complex (typically $s = 1/2 + iy$) and $t \in [0,1]$ interpolates from pure tau-wave ($t = 0$) to pure zeta ($t = 1$).

11.7 Desmos Formulation

The system returns to its initial state when $h \approx 11{,}579.99999998377$, giving $6390 \times h \approx 73{,}996{,}200$.

12 Complex Exponential Sequence Algebra

12.1 Notation

- $n = [0 \ldots Z]$ with $Z = 6390$
- $r = n \cdot e^{i\tau^k}$, $\quad b = \frac{n[0 \ldots (k+1)]}{k} e^{i\tau^k}$, $\quad c = e^{i\tau^n}$
- $a = \sin(n\, e^{i\tau})$, $\quad a_c = \sin(n\, e^{i\tau^{-v}})$, $\quad v = \tau K$

12.2 Key Properties

Observation 1 (Zero-Difference Property)**.** For all valid k ($k \geq 1$ and $k + 1 \leq Z$): $b[k+1] - c[k+1] = 0$.

As k increases, $e^{i\tau^k}$ circles the unit circle with increasing rapidity: $k = 1$ gives 1 rotation, $k = 2$ gives ~ 6.28, $k = 3$ gives ~ 39.5, $k = 4$ gives ~ 248.

13 n-Dependent Exponential Sequence Algebra

13.1 System Definition

$$\sin\big(n \cdot e^{i\tau^k}\big), \quad n = [-N \ldots N], \quad k = n(Z\tau).$$

As Z increases, all points arrange from two spirals to a line between -1 and 1, with a settle point:

$$61.46517051810098 < X_{\text{settle}} < 61.46517051810099.$$

13.2 Behaviour of $\sin(n^{i\tau^k})$

All points converge to $\sin(1) = 0.84147098\ldots$ At 180–600 N with Z near maximum, points distribute approximately 0.01724173 apart.

13.3 Convergence and Crossing Points

At $Z = 0$, increasing N:

- First imaginary crossing: $N_{\text{firstCross}} = 4.810477380965351 = e^{\tau/4}$
- First real crossing: $N_{\text{SecondCross}} = 23.140692632779263 = e^{\tau/2}$
- Period completion: $N = N_{\text{SecondCross}} \cdot Q, \quad Q \approx 111.3178 = e^{3\tau/4}$

Further analysis: $X_{\text{Fixed}} \approx 64.52642412788921$ is potentially where the system fully settles.

13.4 Point Accumulation Pattern

For $\sin(n \cdot e^{i\tau^k})$ at $N = 0$, $Z = 0$: at $N = 0.25$ there are 2 points; at $N = 0.75$, 3 points; at $N = 1.25$, 4 points; etc. Points fall into a circle of radius $\sin(1) \approx 0.84147098$; once full, they transition to a larger shape resembling a cell mid-division, reached when $N = N_{\text{SecondCross}} \cdot Q$.

13.5 Notable Z Values

- $Z = 0.000211085799254870 37$: transition point; increasing by $\tau^{-2}/360$ triggers reconfiguration
- Coalescence requires $N >\sim 3440$–3480 at very small Z, or $Z \approx 0.000172$ at normal scales
- At $Z = 0.000097$, $N = 4000$: the larger (green) shape is targeted

A Full Digit String: $y = 3932$ Initial Digits

The complete initial digit string before the repeating tail 19718309859154929577464788732394366 emerges:

159154943091895335768883763372514362034459645740456448747667344058896797634226535090113802766253085956072842726757958036892911846114578652877967410731699839229239966937409077573077746396925307688717

End of document.